

# MEAN WIND AND POTENTIAL TEMPERATURE PROFILES IN THE ATMOSPHERIC SURFACE LAYER: FURTHER INVESTIGATIONS

Edi Santoso<sup>1</sup>

## Abstract

*The surface layer theory will be presented. Monin – Obukhov similarity theory has played a leading role in most attempts to interpret experimental data on surface layer turbulence. It will also be showed how investigators have modified it for convective conditions. It presents methods for coupling surface layer profiles to profiles higher in the atmospheric boundary layer as well. Most investigators preferred to remain within the classical paradigm that strongly dependent on surface parameters. However, the results based on the classical approach did not merge smoothly into the uniform layer. The new results that considered parameters above surface layer gave better matching to the whole profile.*

## Intisari

*Di dalam tulisan ini akan disajikan perkembangan teori lapisan permukaan. Teori kemiripan Monin – Obukhov memegang peranan penting di dalam setiap usaha untuk menganalisis data turbulen lapisan permukaan dari lapangan. Juga akan dibahas bagaimana para peneliti memodifikasi teori tersebut untuk kondisi atmosfer yang konvektif. Berbagai metoda dikembangkan untuk menggabungkan profil dekat permukaan dengan profil di bagian atasnya. Sebagian besar para peneliti tetap berpegang pada teori klasik yang mempertimbangkan secara kuat parameter-parameter permukaan. Pendekatan lebih baru yang mempertimbangkan parameter-parameter yang ada di lapisan lebih atas memberikan hasil yang lebih baik.*

## 1. INTRODUCTION

The Atmospheric Boundary Layer (ABL) is a layer of air at the bottom of the atmosphere of order 1 to 2 km thick that experiences frictional drag against the surface, and experiences a diurnal cycle of temperature change in response to changing surface conditions (Stull 1988). This layer is often turbulent, and is capped by a statically stable layer (e.g., the entrainment zone in Fig. 1) that separates the turbulent boundary layer from the rest of the less-turbulent troposphere. During daytime over land, or any time when a warmer surface underlies cooler air, vigorous convective circulations form in the boundary layer as warm air rises in the form of large diameter thermals. Thermal diameters are roughly the same size as the convective ABL depth. This type of boundary layer is called a convective ABL or a

convective mixed layer (ML). The bottom part of this layer is the focus of this paper.

One can identify subdomains of the convective ABL that have different similarity scalings. The term mixed layer (ML) is used here to represent the whole convective boundary layer that is nonlocally statically unstable (Stull, 1991), and which is undergoing vigorous convective overturning associated with coherent rising thermals. Fig. 1a identifies layers in the convective ML, using wind speed as an example. The average ML depth is  $z_i$ .

In the middle of the ML is a deep region of vertically-uniform wind speed (Fig. 1a), wind direction, and potential temperature (Fig. 1b), called the uniform layer (UL). In the UL, wind speed ( $M$ ) and direction are nearly uniform, but subgeostrophic, with height ( $z$ ). The wind is subgeostrophic because thermals communicate surface drag information via nonlocal transport.

<sup>1</sup> UPT. Hujan Buatan-BPP.Teknologi, Jl. M.H.Thamrin No. 8, Jakarta 10340

The profiles are vertically uniform because of the intense mixing taking place there.

At the bottom of the ML is the surface layer (SL), the nearly uniform flux region where Monin-Obukhov (MO) similarity theory applies (e.g., Businger et. al., 1971; Dyer, 1974). In this SL, the wind speed is nearly logarithmic with height, dominated by mechanically-generated small-eddy turbulence within the wall shear flow (Stull, 1997).

At the very bottom of the SL are a roughness sublayer and blending layer, where the wake turbulence immediately behind individual roughness elements, gradually blends into a horizontally uniform turbulence field as height increases. These layers have depth of the same order as the roughness elements (centimeters for blades of grass, to tens of meters for trees), but are not shown in Fig. 1.

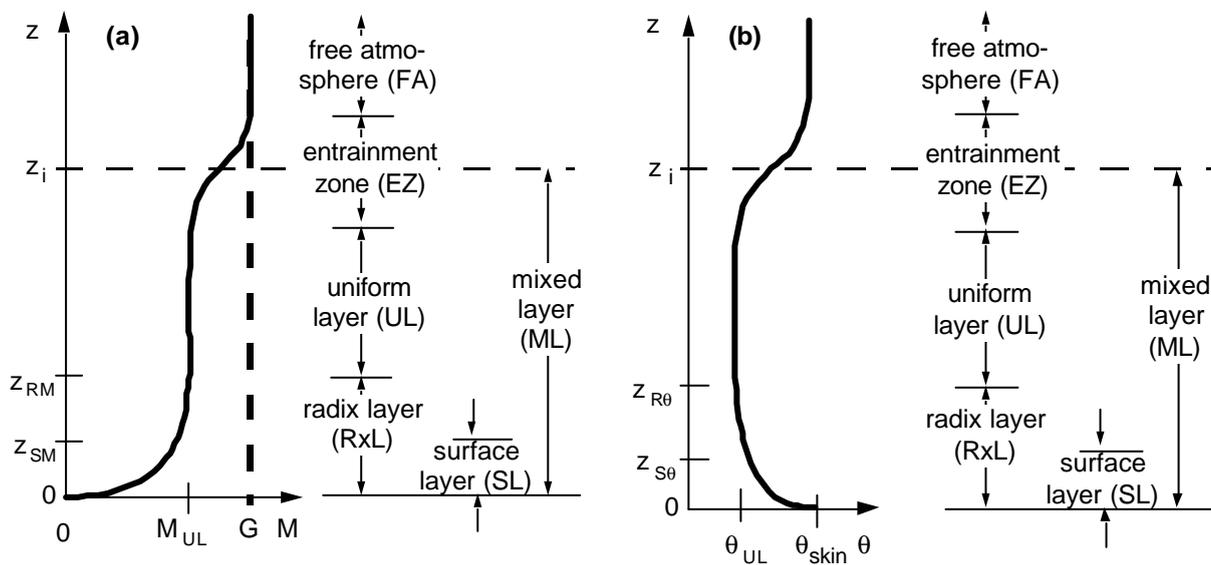
There is no precise definition of the SL. Qualitatively, the SL is that part of the convective ML immediately above the surface where vertical variations of vertical heat, momentum and moisture fluxes can be ignored. This simplifies the theory by allowing flux variations to be neglected. It also constrains its applicability only to the bottom 10% or less of the ML, assuming linear vertical profiles of heat and momentum fluxes.

This paper presents a SL theory, and shows how investigators have modified it for convective conditions. It also presents methods for coupling SL profiles to profiles higher in the ABL.

## 2. BUCKINGHAM PI ANALYSIS

Many of the papers reviewed utilize a technique known variously as similarity theory, dimensional analysis, or Buckingham Pi analysis (Stull, 1988). This method identifies variables that might be relevant to a flow situation, and forms them into dimensionless groups called Pi groups. The relationship between Pi groups is not given by the theory, but must be found empirically. However, the result has the potential of being universal. For such universal behavior, data from many different sites obtained during different conditions will collapse into a single line or curve. Hence, the data is said to be self-similar when plotted in this dimensionless form. An equation designed to describe this empirical similarity curve is said to be a similarity relationship. MO theory, to be reviewed in the next section, is an example of a similarity relationship for the profiles in the SL.

Following the statistical turbulence conventions of Reynolds (1895), average or mean values will be indicated with an overbar ( $\bar{\phantom{x}}$ ), and instantaneous deviations from the mean will be indicated with a prime ( $\prime$ ). The average is typically found as a 30-minute average at one site, or as a line or area average measured by aircraft or remote sensors.



**Fig. 1.** Idealized vertical profiles of (a) wind speed  $M$  and (b) potential temperature  $q$  in the atmospheric boundary layer.  $G$  represents the geostrophic wind speed,  $q_{skin}$  is the potential temperature of the surface skin, and  $z_i$  is the mixed-layer depth. The subscripts are  $UL$  for the uniform layer,  $R$  for the radix layer,  $S$  for the surface layer,  $q$  for potential temperature, and  $M$  for wind speed.

### 2.1. Framework for Monin-Obukhov Similarity Theory

Early wind-tunnel measurements for aerodynamically rough flow suggested that the mean wind shear,  $\partial \overline{M} / \partial z$ , varies with vertical distance from the wall of the wind tunnel,  $z$ , according to:

$$\frac{\partial \overline{M}}{\partial z} = \frac{u_*}{k \cdot z} \tag{1}$$

where  $u_*$  is the surface friction velocity (i.e., the square root of the magnitude of kinematic momentum flux at the surface) and  $k$  is von Karman's constant (approximately 0.4). This situation applies for conditions with no vertical heat flux, that is, for statically neutral flows. This shear equation is also anticipated using dimensional analysis. Eq. (1) lies at the heart of all treatments of neutral atmospheric surface layers, and takes advantage of the characteristic that shear flows have predominantly small-size eddies, thereby causing local transport.

One can integrate (1) to yield a logarithmic profile of wind with height:

$$\overline{M}(z) = \frac{u_*}{k} \cdot \ln\left(\frac{z}{z_o}\right) \text{ for } z \geq z_o \tag{2}$$

where  $z_o$  is a constant of integration that is interpreted as a surface aerodynamic roughness length. It is the height at which  $\overline{M}$  extrapolates to zero assuming zero displacement height.

Eq. (2) is applicable only for flow over an aerodynamically rough surface, which implies that molecular viscosity can be neglected. Flow is considered to be aerodynamically rough (Sutton, 1953; Businger, 1973; Azad, 1993) if:

$$\frac{u_* \cdot z_o}{\nu} > 2.5 \tag{3}$$

where  $\nu$  is the kinematic viscosity of air. In the atmosphere, where  $\nu = 1.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$  and where typical values of  $u_*$  are of order  $0.1 \text{ m s}^{-1}$ . Eq. (3) implies that  $z_o$  must be greater than  $3.75 \times 10^{-4} \text{ m}$  in order to be considered a fully rough surface.

For situations where surface roughness elements are so close together as to form a dense canopy, the flow skims over the canopy with a log-profile that is displaced distance  $z_d$  from the ground:

$$\overline{M}(z) = \frac{u_*}{k} \cdot \ln\left(\frac{z - z_d}{z_o}\right) \text{ for } z \geq z_d + z_o \tag{4}$$

The aerodynamic roughness length  $z_o$  is interpreted as a property of the surface. Eq. (4) indicates that for different values of  $z_o$  and  $u_*$ , there is a family of curves describing a vertical distribution of wind speed. As described at the beginning of this paper, all these curves are similar, meaning that when the height is scaled by  $z_o$  and the wind speed is scaled by  $u_*$ , they collapse into one curve. This is the value of similarity theory.

The presence of a vertical turbulent heat flux introduces new similarity parameters and modifies the shape of the wind profile away from logarithmic. Monin and Obukhov (1954) considered the effect of stability and proposed a similarity theory for a stratified SL, known as MO similarity theory. MO similarity theory has been the favored tool for finding wind and potential temperature profiles in the SL.

According to this theory, mean wind and temperature profiles in the SL are a function of height  $z$ , surface kinematic shear stress,  $\tau$  (sometimes surface friction velocity  $u_* = \sqrt{|\tau|/\rho}$ , is used as a surrogate for stress), surface kinematic heat flux,  $\overline{w'q'_s}$  (or virtual potential temperature flux  $\overline{w'q'_{v,s}}$ , which includes the moisture contribution to buoyancy), and buoyancy parameter,  $g/T_v$ , where  $\rho$  is air density,  $g$  is gravitational acceleration,  $T_v$  is virtual absolute temperature near the surface,  $w$  is vertical velocity, and subscript  $s$  indicates near surface conditions. Therefore, the vertical profiles of mean wind and potential temperature in the SL may be written in the form of a product of a dimensional constant factor and a dimensionless universal function of argument  $z = z/L$ , where the Obukhov length  $L$  is:

$$L = - \frac{u_*^3}{k \cdot \frac{g}{T_v} \cdot \overline{w'q'_{v,s}}} \tag{5}$$

MO similarity theory is strongly dependent on the surface roughness and on static stability, but is virtually independent of factors higher in the ABL. Within this theory, dimensionless wind shear and potential temperature gradients for the unstable SL as a function of dimensionless height  $z/L$  are defined as:

$$f_m\left(\frac{z}{L}\right) \equiv \frac{k \cdot z}{u_*} \cdot \frac{\partial \overline{M}}{\partial z} \tag{6a}$$

$$f_t\left(\frac{z}{L}\right) \equiv \frac{z}{q_*} \cdot \frac{\partial \overline{q}}{\partial z} \tag{6b}$$

where  $L$  is negative for statically unstable conditions, and  $q_* = -w'q'_s / (k \cdot u_*)$  is a SL scaling temperature.

These non-dimensional expressions (6a and 6b) can be integrated formally without commitment about the exact forms of the non-dimensional functions, to yield the mean wind and potential temperature profiles:

$$\overline{M}(z) = \int_{z''=z_{om}}^z \frac{u_*}{k \cdot z''} \cdot f_m \left( \frac{z''}{L} \right) dz'' \quad (7a)$$

$$\overline{q}(z) = \int_{z''=z_{ot}}^z \frac{q_*}{z''} \cdot f_t \left( \frac{z''}{L} \right) dz'' \quad (7b)$$

where  $z_{om}$  and  $z_{ot}$  are surface roughness lengths for wind and potential temperature, respectively, and  $z''$  is a dummy variable of integration. The function  $f_m$  varies away from unity as the static stability varies from neutral, and there is analogous variation of  $f_t$ . However, these functional forms are not known from first principles, but must be estimated empirically.

These integrated relationships are sometimes very complicated to solve analytically from the original gradient forms. There is also uncertainty about the difference between the two roughness lengths  $z_{om}$  and  $z_{ot}$ . Therefore, the study of the integrated profiles from the dimensionless gradient forms needs special consideration.

To date, MO similarity theory has played a leading role in most attempts to interpret experimental data on SL turbulence. Because it has proved quite successful in the analysis of SL mean flow, it has virtually been accepted as a paradigm.

### 2.2. Empirical Estimates of the MO Profile Functions $f$

There are many published non-dimensional gradient forms for wind and potential temperature for the unstable SL. Dyer (1974) published a survey of existing flux-profile relationships for wind, temperature and humidity. Similar surveys later were given by Yamamoto (1975) and Yaglom (1977). The most recent ones were published by Hogstrom (1988), Sorbjan (1989) and Zilitinkevich (1991). The various derivations for convective SL are described briefly in this section.

In early derivations of non-dimensional profiles for statically unstable conditions, a log-linear approximation was common (i.e., Monin and Obukhov, 1954; Sheppard, 1958). The function  $f_m$

can be expanded as a power series and its first-order approximation can be written as:

$$f_m \left( \frac{z}{L} \right) = f_m(0) + g \cdot \frac{z}{L} + \dots \approx 1 + g \cdot \frac{z}{L} \text{ for } z < |L| \quad (8a)$$

where  $f_m(0) = 1$  and  $g$  is an empirical constant. A similar approximation for potential temperature was:

$$f_t \left( \frac{z}{L} \right) \approx \frac{K_m}{K_H} \cdot \left( 1 + g \cdot \frac{z}{L} \right) \text{ for } z < |L| \quad (8b)$$

where  $K_m$  and  $K_H$  are turbulent exchange coefficients for momentum and heat, respectively. The reported values of  $g$  and  $K_m / K_H$  in unstable surface layer vary from author to author (Panofsky et al., 1960; Taylor, 1960; Deacon, 1962; Zilitinkevich and Chalikov, 1968a; Webb, 1970). These log-linear equations are generally only applicable for a very limited range of  $z/L$ .

Ellison (1957), Yamamoto (1959) and Panofsky (1961) independently derived the structure of wind and temperature by a unified equation that covered a broader range of stability. Such a semi empirical equation is of the form:

$$f^d \left( \frac{z}{L} \right) + g \cdot \frac{z}{L} \cdot f^d \left( \frac{z}{L} \right) = 1 \quad (9)$$

where  $g$  is an empirical constant that is not necessarily the same for wind and potential temperature, and which also varies from author to author (Panofsky et al., 1960; Paulson, 1970; Businger et al., 1971; Carl et al., 1973; Yamamoto, 1975). A variant of this equation was known as the KEYPS profile (Lumley and Panofsky, 1964).

Other authors presented a unified equation for wind and temperature derived semi-empirically as power laws of the form:

$$f \left( \frac{z}{L} \right) = a \cdot \left( \frac{z}{L} \right)^{-d} \quad (10)$$

where  $a$  and  $d$  are empirical constants that might be different for wind and potential temperature. The reported values of  $a$  and  $d$  are different from author to author (Taylor, 1960; Swinbank, 1968; Zilitinkevich and Chalikov, 1968a; Foken and Skeib, 1983), but generally the power  $d$  equals 1/3 for both wind and temperature.

One derivation of the power law equation was from a modified free convection theory (Priestley, 1955), which predicted that wind and temperature profiles are proportional to  $z^{-1/3}$ . Free convection is a state of the boundary layer in which vertical transfer of heat and momentum by mechanical

turbulence can be neglected compared to that by buoyant convection.

Most widely reported non-dimensional wind and temperature profiles for free convection are modified logarithmic forms:

$$f\left(\frac{z}{L}\right) = a \left(1 - g \cdot \frac{z}{L}\right)^{-d} \quad (11)$$

where  $a$ ,  $g$  and  $d$  are arbitrary constants that can differ for wind and potential temperature, and that also vary from author to author [Dyer, 1967; Dyer and Hicks, 1970; Paulson 1970; Businger et al., 1971; Carl et al., 1973; Pruitt et al., 1973; Dyer, 1974; Kondo (see Yamamoto, 1975); Lettau, 1979; Dyer and Bradley, 1982; Korrell et al., 1982; Kai, 1982; Fukui et al., 1983 (see Zilitinkevich, 1991); Panin and Piazena, 1983 (see Zilitinkevich, 1991); Foken and Skeib, 1983; Hogstrom, 1988; Frenzen and Vogel, 1992; Oncley et al., 1996]. The reported values of  $a$  are usually near 1 for wind but vary for temperature, while the reported values of  $d$  range over 1/3 to 1/4 for wind, and 1/3 to 1/2 for temperature.

Perhaps the most frequently cited form of (11) is the Businger - Dyer relationship (Businger et al., 1971; Dyer, 1974; Hogstrom, 1988; Oncley et al., 1996), which for statically unstable conditions was originally proposed to be of the form:

$$f_m\left(\frac{z}{L}\right) = \left(1 - 15 \frac{z}{L}\right)^{-1/4} \quad (12a)$$

$$f_t\left(\frac{z}{L}\right) = 0.74 \left(1 - 9 \frac{z}{L}\right)^{-1/2} \quad (12b)$$

based on SL data collected during the 1968 Kansas field experiment (Izumi, 1971). These equations were based on a von Karman constant of  $k = 0.35$ , which was later found to be too small. When these dimensionless forms are integrated to give mean wind or potential temperature with height, the results are quite complex. For example the integrated wind speed equation in the statically unstable SL (Paulson, 1970; Stull, 1988) is:

$$\overline{M} = \frac{u_*}{k} \left[ \ln\left(\frac{z}{z_o}\right) - 2 \ln\left(\frac{1+x}{2}\right) - \ln\left(\frac{1-x^2}{2}\right) + 2 \tan^{-1}(x) - \frac{p}{2} \right] \quad (2.13)$$

where  $x = (1 - 15z/L)^{-1/4}$ , assuming  $z_o/L$  is negligible. Hogstrom (1988) proposed a version of (11) with different constants, to utilize the generally accepted value of  $k = 0.4$  for the von Karman constant.

All of the semi-empirical equations above were mainly derived using data collected from short masts in the bottom of the SL with limited range of  $z/L$ . Kader and Perepelkin (1984, 1989), Kader (1988), Kader and Yaglom (1990), and

Zilitinkevich et al. (1997, 1998) proposed non-dimensional equations for wind and potential temperature profiles that are applicable for the whole SL. They applied two- or three-layer models to match the whole SL. Their equations are similar to (8) for the lowest layer, and to a modified free convection (11) or to the unmodified free convection equation (10) for middle and upper layers.

Since the early 1950s, when modern observation techniques became available, many experiments have been conducted to study the profiles. Most of the results were systematically interpreted within the framework of MO similarity theory.

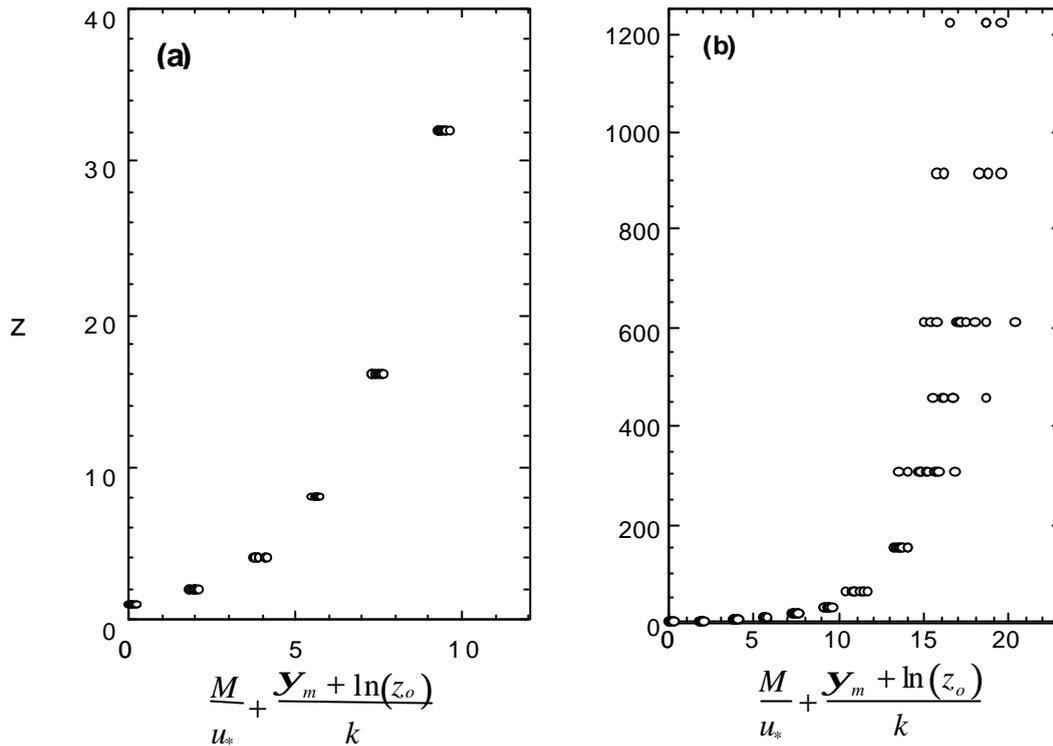
### 2.3. Performance of Classical MO Theory vs. Altitude

Most classical SL similarity equations are strongly dependent on surface parameters, but are virtually independent of factors higher in the ML. Typically lacking is dependence on ML depth, temperature within the UL, winds within the UL, and geostrophic wind speed. For this reason, one cannot expect the SL equations to merge smoothly into the UL, because no information about the UL is included in those equations. Panofsky (1978) points out that convective-matching-layer and free-convection-layer formulations (Priestley, 1955; Kaimal et al., 1976) fail near the bottom of the UL, where the shear and potential-temperature gradient approach zero.

This situation is illustrated in Fig. 2, where the abscissa has been normalized according to Businger et al. (1971) - Dyer (1974) similarity theory. In this normalization, all the data will collapse to a single curve regardless of static stability in those regions where SL similarity theory is valid. While Fig. 2a shows that SL similarity works well in the bottom 40 m of the ML for the Minnesota data set (to be described in more detail later), Fig. 2b shows that SL theory is less successful higher in the RxL and in the UL. Namely, the classical surface layer does not extend up to the base of the UL for these data, resulting in data points that do not collapse onto a single curve.

### 3. Modification of MO Similarity to Improve Performance at Altitudes above the SL

While new results (Santoso, 1997; Santoso and Stull, 1998) proposes completely new profile equations to the bottom half of the ML, most previous investigators preferred to remain within the classical paradigm of SL theory when trying to explain wind and temperature profiles higher



**Fig. 2.** Wind profiles for all 11 runs of the Minnesota campaign. Abscissa is normalized using surface-layer similarity, where  $z_o$  is aerodynamic roughness length,  $k = 0.4$  is von Karman's constant, and  $y_m$  is the integrated wind profile stability-correction function of Businger - Dyer. (a) Within the SL. (b) Within the RxL and UL.

above the ground. Two approaches have been taken in the past by other investigators: (1) using two separate profiles, one for the SL and one for the mid ABL, and matching the profiles at some intermediate height; and (2) modifying the MO dimensionless profile functions ( $f$ ) to include additional physics.

### 3.1. Profile Matching

It is often necessary to be able to approximate the surface stress and fluxes in terms of mean variables at the grid points in numerical weather prediction models. In some models the lowest grid point is generally well above the surface layer, making it impossible to directly use the SL flux-profile relationships described above. The SL similarity profiles are obviously not applicable at extremely large values of  $z/z_o$ . On the other hand, upper-layer similarity profiles based on geostrophic departure concepts cannot be extrapolated down to low values of  $z/z_o$  due to the very steep gradient of the profiles close to the ground. By matching SL profiles to other profiles for the mid-boundary layer, investigators attempted to relate surface fluxes to conditions

higher in the boundary layer. To do this, it must be assumed that there exists a level in the atmosphere where both lower and upper profiles can be matched. Unfortunately, often this matching height is found to be above the valid range for MO theory.

When this matching procedure is performed using the Businger - Dyer equations for wind and potential temperature profiles in the SL (Eq. 12), one finds the following set of equations (Stull, 1988):

$$\frac{u_b}{u_*} = \frac{1}{k} \cdot \left[ \ln\left(\frac{h_b}{z_o}\right) - A\left(\frac{h_b}{L}\right) \right] \tag{14a}$$

$$\frac{v_b}{u_*} = -\frac{1}{k} \cdot B\left(\frac{h_b}{L}\right) \cdot \text{sign}(f_c) \tag{14b}$$

$$\frac{\Delta q_s}{q_*} = \left[ \frac{K_m}{k \cdot K_H} \right] \cdot \left[ \ln\left(\frac{h_b}{z_o}\right) - C\left(\frac{h_b}{L}\right) \right] \tag{15}$$

where  $A$ ,  $B$  and  $C$  are hopefully "universal" functions;  $u_b$  and  $v_b$  are characteristic horizontal wind scale components;  $h_b$  is a characteristic

thickness or height scale;  $\Delta \mathbf{q}_s = \mathbf{q}_s - \mathbf{q}_b$ , is the potential temperature difference between the surface and the air; and  $f_c$  is the Coriolis parameter.

The thickness or height scale and horizontal wind scale components have been given different definitions depending on the specific implementation of the approach. The height scale  $h_b$  has been defined variously as proportional to the thickness of an Ekman layer,  $u_* / f_c$  (e.g., Gill, 1968; Hess, 1973; Sudararajan, 1975; Arya, 1975; Arya and Wyngaard, 1975; Emeis and Zilitinkevich, 1991); the depth of the mixed layer or the bottom of the overlying inversion layer,  $z_i$  (e.g., Deardorff, 1972; Zilitinkevich and Deardorff, 1974; Yamada, 1976; Brutsaert and Sugita, 1991); or the depth of convective mixing,  $h_c$  (e.g., Clarke, 1970; 1972; Garratt and Francey, 1978).

The horizontal wind scale components  $u_b$  and  $v_b$  were initially taken as the surface geostrophic wind components  $u_g$  and  $v_g$  (e.g., Deacon, 1973). Recognizing the uncertainties involved in measurements of geostrophic winds, some investigators (e.g., Zilitinkevich, 1969; Clarke and Hess, 1974; Melgarejo and Deardorff, 1974) proposed the values of the wind velocity components,  $u_h$  and  $v_h$ , measured at height  $z$  equal to some fraction of  $u_* / f_c$ . Others suggested the values of the wind velocity components measured at height  $z$  equal to  $z_i$  (e.g., Zilitinkevich and Deardorff 1974); the geostrophic wind averaged over the convective boundary layer,  $u_{ga}$  and  $v_{ga}$  (e.g., Arya and Wyngaard, 1975; Yamada, 1976); or the observed wind velocity components averaged over the atmospheric boundary layer,  $u_a$  and  $v_a$  (e.g., Deardorff, 1972; Arya, 1977; Garratt et al., 1982). Considering that under convective conditions the cross-geostrophic component of wind velocity is usually very small, some authors (e.g., Brutsaert and Sugita, 1991; Sugita and Brutsaert, 1992) proposed the observed wind velocity averaged over the mixed layer,  $V_a$ . Similar approaches have been proposed in the literature for the potential temperature  $\mathbf{q}_b$ . The relations involving the layer-averaged parameters appeared to give better results in calculating the similarity functions.

There were many theoretical attempts to calculate the "universal" functions  $A$ ,  $B$  and  $C$ . Some were based on matching analytical solutions of upper-layer equations with surface similarity profiles (e.g., Zilitinkevich and Chalikov, 1968b; Brown, 1978). Most of the derivations of the similarity functions were based on experimental data (e.g., Clarke, 1970; 1972; Deacon, 1973; Clarke and Hess, 1974; Arya, 1975; Brutsaert and

Sugita, 1991). They all demonstrated a great scatter of data, partly because the hoped-for universality of  $A$ ,  $B$  and  $C$  was not realized. Also, profile-matching theory was designed for an idealized situation that was rarely found in the real atmosphere.

The functions  $A$ ,  $B$  and  $C$  are generally accepted to depend on several variables; however, the dependence on  $(h_b / L)$  is the only one that has been practical or useful. The other ones that have received most attention in the literature are baroclinicity or the effect of thermal wind; the ratio of convective and rotational height scales  $(f_c \cdot z_i / u_*)$ ; nonstationarity or diurnal heating effects; inertia, large-scale advection; momentum entrainment; and large scale vertical velocity or subsidence. Magnitudes of these effects are not easily calculated from experimental data, resulting in considerable error.

While steady progress was made in the analysis of turbulent transport phenomena in the upper region of the convective boundary layer, there was still no general agreement on robust and reliable similarity functions relating profiles of mean wind and potential temperature near the surface to those in the upper region of the ML. Unlike the surface layer, conditions higher in the ML are rarely the result of local small-eddy equilibrium. Moreover, the available experimental data for these upper layers were relatively rare and often lacked the statistical robustness of the SL observations.

### 3.2. Modification of MO Profile Equations

Some SL similarity equations have been modified to apply higher in the ML. Kader and Perepelkin (KP, 1989) proposed a "continuous" formula to approximate the shape of a discontinuous three-layer model for the wind shear in the bottom half of the ML (their Eq. 11):

$$\frac{z}{u_*} \frac{\partial \overline{M}(z)}{\partial z} = 2.5 \left[ \frac{\{1 - 0.1(z/L_1)^2\}^{1/3}}{(1 - 3z/L_1)} \right] \quad (16)$$

where  $L_1 = k \cdot L$ , and with a von Karman constant of  $k = 0.4$ . Sorbjan (ZS, 1986) suggested the following (his Eq. 38) for wind shear in the bottom half of the ML:

$$\frac{k \cdot z_i}{w_*} \frac{\partial \overline{M}(z)}{\partial z} = 0.42 \frac{(1 - \alpha/z_i)^{2/3}}{(z/z_i)^{4/3} (-z_i/L)^{2/3}} \quad (17)$$

where  $a$  is a constant equal to 1.5. A von Karman constant of  $k = 0.35$  was used in this equation (Sorbjan 1986).

Fig. 3 compares these two relationships to the Minnesota data of period 7C1 (see Santoso and Stull, 1998 for more detail on the Minnesota measurement periods or "runs"). Also plotted is an extension of the Businger - Dyer (BD) profile (Eq. 12a) above the surface layer, because this is arguably the most-used profile equation in the meteorological literature. For comparison, also plotted is the radix-profile equation (Santoso and Stull, 1998).

Of the three non-radix relationships plotted in Fig. 3, the proposal by Sorbjan appears to work the best above the top of the SL; however, even it has substantial errors for all Minnesota runs. Of all the MO relationships discussed, the Sorbjan relationship is the most successful at higher altitudes, and will be examined in more detail in the next section.

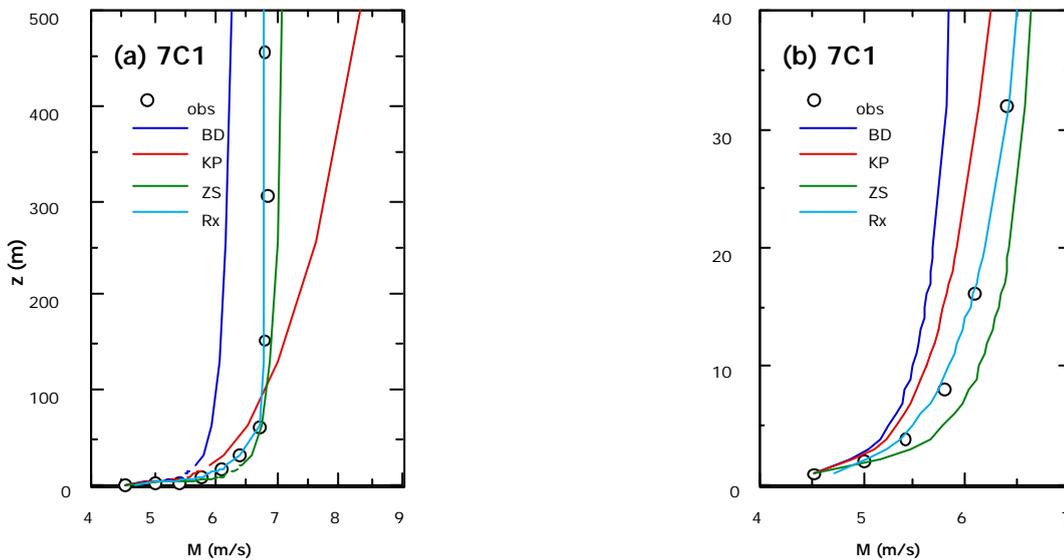
### 4. WIND PROFILES VS. SHEAR PROFILES

Most of the similarity relationships described earlier for the surface layer are given as a dimensionless wind shear  $f_m$  or potential-temperature gradient  $f_t$  (see Eqs. 6). During this

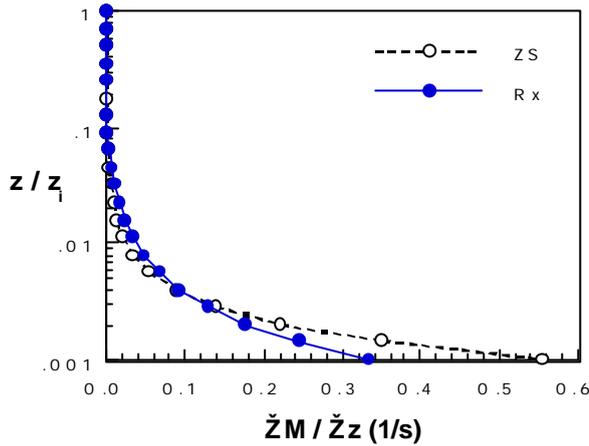
research, it became apparent that inaccuracies of wind-profile relationships are hidden when profiles are expressed as shears or gradients. However, when those profiles are integrated with height to get the actual wind speeds, the errors are revealed, and can accumulate to cause substantial discrepancies between the observed wind speed and the parameterized profile. This is unfortunate, because for many practical purposes such as wind loads on structures, pollutant transport, and wind power generation, it is the speed and not the shear that is needed.

As an example, Fig. 4 compares the wind shear using the Sorbjan (ZS, 1986) relationship versus the RxL relationship (Santoso and Stull, 1998). The difference between these two curves is subtle; it is not easy to discriminate between the two relationships. However, when integrated over height to get wind speed, the deficiencies of the ZS relationship become apparent and significant when compared to RxL theory profiles (see Fig. 3). The RxL wind speeds are more accurate over a wider range of heights than the ZS speeds.

One might argue that this is an unfair test, because integrating up from a small wind near the surface, such as from zero wind at the roughness height, might amplify small initial errors. To examine this argument one can recompute the height integration, but starting at different heights. This process is repeated for each data point in the observed wind profile, generating a set of curves



**Fig. 3.** Mean wind observations (obs) from the Minnesota field experiment Run 7C1, compared to three SL models [KP = Kader and Perepelkin, 1989; ZS = Sorbjan, 1986; BD = Businger - Dyer (Businger et al., 1971); Rx = radix (Santoso and Stull, 1998)]. (a) Above, and (b) within the traditional SL.



**Fig. 4.** Comparison of wind-shear profiles from Sorbjan (ZS, Eq. 17) using modified MO theory, and from RxL theory, for typical conditions at Minnesota where  $z_i = 2000$  m,  $z_{Rm} = 200$  m,  $L = -20$  m,  $\overline{M}_{UL} = 10$  m s<sup>-1</sup>,  $w_s = 2$  m s<sup>-1</sup>. The differences are subtle, but significant when integrated over height to get wind speed.

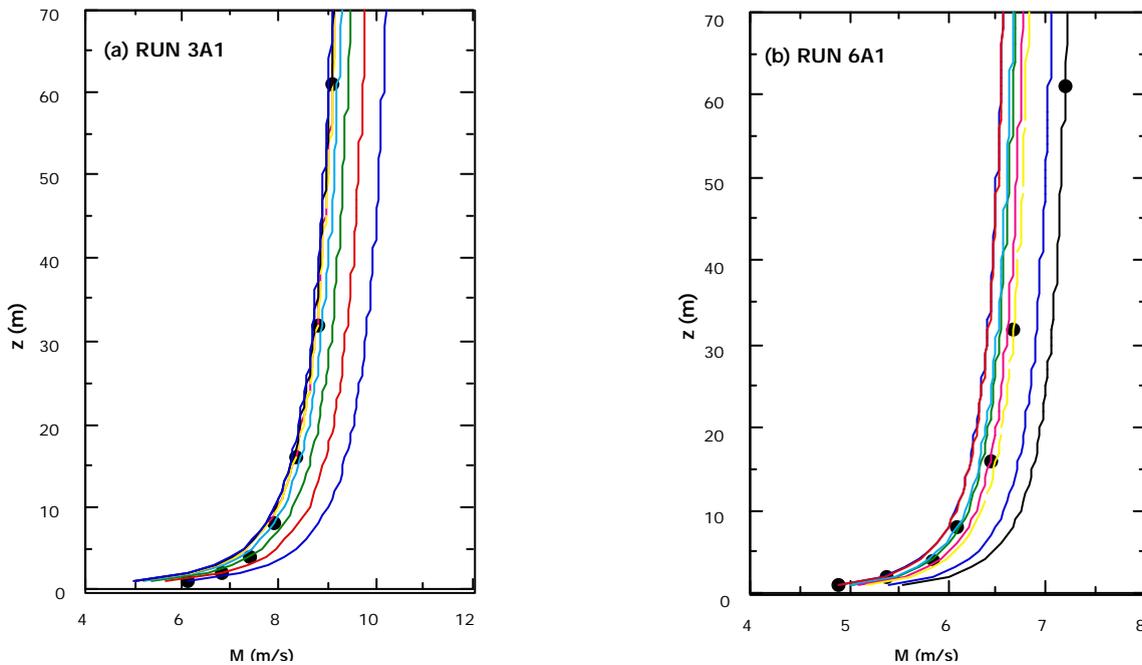
Examples of the integrated ZS wind speed profiles are shown in Fig. 5, for Minnesota runs 3A1 and 6A1. Most of the curves do not lie on top of each other. Furthermore, the direction of the error is not consistent: run 3A1 has less shear than the integrated ZS curves, while run 6A1 has more shear. When this exercise is repeated for each of the Minnesota runs, magnitudes of ZS wind-speed errors of roughly  $\pm 3.0$  m s<sup>-1</sup> for runs 2A1, 2A2;  $\pm 1.0$  m s<sup>-1</sup> for runs 3A1 and 3A2;  $\pm 0.7$  m s<sup>-1</sup> for runs 6A1, 6A2, 6B1, and 7C2; and  $\pm 0.3$  m s<sup>-1</sup> for runs 7C1 and 7D1 were found. This compares to wind speed errors of  $\pm 0.3$  m s<sup>-1</sup> or less for all runs using the radix-layer equation.

Thus, it appears that wind speed gives a more sensitive test of the accuracy of a similarity relationship than does shear. It is recommended that future proposals for similarity relationships be tested in their integrated form, such as wind-speed profiles. These plots also show that while the theory of Sorbjan is the best of the MO theories, it still leaves room for improvement.

### 5. SUMMARY AND CONCLUSIONS

such that each curve exactly passes through one point. If the profile similarity theory is valid, then all of the curves should lie nearly on top of each other.

Most SL similarity theories discussed earlier in this paper are based on the following premises:  
 1. Turbulent flux is approximately uniform with height (constant flux  $\pm 10\%$ ),



**Fig. 5.** Observed wind speeds ( $M$ ) [points], and profile relationships [curves] found by integrating the Sorbjan's (1986) equation up and down from each observed wind speed. Namely, a separate profile curve is calculated for each data point. If Sorbjan's equation had been accurate, then all of the curves would have coincided.

2. Turbulence consists of "small eddies", causing local transport,
3. Turbulence is predominantly generated mechanically by shear flow near the ground with minor modifications for static stability,
4. Feedback exists between the mean flow and the dominant eddies.

The first premise simplifies the theory by allowing flux variations to be neglected, but it constrains the depth of applicability to the bottom 10% of the ML, assuming heat flux decreases roughly linearly with height during near-free convection.

The second premise suggests that ML depth  $z_i$  should not be relevant; again making it unlikely that the theory could be successful higher in the ABL.

The third premise implies that surface roughness length  $z_o$  is important, which is indeed the case close to the ground.

The fourth premise has the following interpretation for flow very near the bottom boundary. Turbulence transports momentum; momentum-flux divergence alters the mean-wind profile; and shear in the mean-wind profile generates small-eddy turbulence. The feedback loop is closed, at least for shear-driven surface layers. This is a fundamental, but infrequently discussed, premise underlying SL similarity theory.

In the nearly free-convective ML, such a feedback is broken. Turbulence still transports momentum; and momentum-flux divergence alters the mean wind profile. However, the mean wind profile does not generate the large-eddy turbulence. Instead, surface heating generates the large, coherent thermal structures. Therefore, the R<sub>x</sub>L equations that considered parameters above the SL can fit the profiles better.

## References

- Arya, S. P. S., 1975: Geostrophic drag and heat transfer relations for the atmospheric boundary layer. *Quart. J. Roy. Meteor. Soc.*, **101**, 147 - 161.
- Arya, S. P. S., 1977: Suggested revisions to certain boundary layer parameterization schemes in atmospheric circulation models. *Mon. Weather. Rev.*, **105**, 215 - 227.
- Arya, S. P. S. and J. C. Wyngaard, 1975: Effect of baroclinicity on wind profiles and the geostrophic drag law for the convective planetary boundary layer. *J. Atmos. Sci.*, **32**, 767 - 778.
- Azad, R. S., 1993: *The Atmospheric Boundary Layer for Engineers*. Kluwer Academic Publishers, Dordrecht, The Netherlands. 565 pp.
- Brutsaert, W. and M. Sugita, 1991: A bulk similarity approach in the atmospheric boundary temperature to determine regional surface fluxes. *Bound-Layer Meteor.*, **55**, 1 - 23.
- Businger, J. A., 1973: Turbulence transfer in the atmospheric boundary layer. In *Workshop on Micrometeorology* (Ed. D. A. Haugen). American Meteorological Society, Boston, 67 - 98.
- Businger, J. A., J. C. Wyngaard, Y. Izumi and E. F. Bradley, 1971: Flux-profile relationships in the atmospheric surface layer. *J. Atmos. Sci.*, **28**, 181 - 189.
- Carl, D. M., T. C. Tarbell and H. A. Panofsky, 1973: Profiles of wind and temperature from towers over homogeneous terrain. *J. Atmos. Sci.*, **30**, 788 - 794.
- Clarke, R. H., 1970: Observational studies in the atmospheric boundary layer. *Quart. J. Roy. Meteor. Soc.*, **96**, 91 - 114.
- Clarke, R. H., 1972: Discussion of "Observational studies in the atmospheric boundary layer". *Quart. J. Roy. Meteor. Soc.*, **98**, 234 - 235.
- Clarke, R. H. and G. D. Hess, 1974: Geostrophic departure and the functions A and B of Rossby-similarity theory. *Bound.-Layer Meteor.*, **7**, 267 - 287.
- Deacon, E. L., 1962: Aerodynamic roughness of the sea. *J. Geophys. Res.*, **67**, 3167 - 3172.
- Deacon, E. L., 1973: Geostrophic drag coefficients. *Bound.-Layer Meteor.*, **5**, 321 - 340.
- Deardorff, J. W., 1972b: Parameterization of the planetary boundary layer for use in general circulation models. *Mon. Weather. Rev.*, **100**, 93 - 106.
- Dyer, J. A., 1974: A review of flux-profile relationship. *Bound.-Layer Meteor.*, **7**, 363 - 372.
- Dyer, J. A., 1967: The turbulent transport of heat and water vapor in an unstable atmosphere. *Quart. J. Roy. Meteor. Soc.*, **93**, 501 - 508.
- Dyer, J. A. and B. B. Hicks, 1970: Flux-gradient relationships in the constant flux layer. *Quart. J. Roy. Meteor. Soc.*, **96**, 715 - 721.
- Dyer, J. A. and E. F. Bradley, 1982: An alternative analysis of flux-gradient relationships at the 1976 ITCE. *Bound.-Layer Meteor.*, **22**, 3 - 19.
- Ellison, T. H., 1957: Turbulence transport of heat and momentum from an infinite rough plane. *J. Fluid Mech.*, **2**, 456 - 466.
- Emeis S and S. S. Zilitinkevich, 1991: Resistance law, effective roughness length, and deviation angle over hilly terrain. *Bound.-Layer Meteor.*, **55**, 191 - 198.
- Foken, Th. and G. Skeib, 1983: Profile measurements in the atmospheric near-surface layer and the use of suitable universal functions for the determination of the turbulent energy exchange. *Bound.-*

- Layer Meteor.*, **25**, 55 - 62.
- Frenzen, P. and C. A. Vogel, 1992: The turbulent kinetic energy budget in the atmospheric surface layer: A review and experimental reexamination in the field. *Bound.-Layer Meteor.*, **60**, 49 - 76.
- Garratt, J. R., J. C. Wyngaard and R. J. Francey, 1982: Winds in the atmospheric boundary layer – prediction and observation. *J. Atmos. Sci.*, **39**, 1307 - 1316.
- Garratt, J. R., 1978: Flux-profile relationships above tall vegetation. *Quart. J. Roy. Meteor. Soc.*, **104**, 199 - 212.
- Garratt, J. R. and R. J. Francey, 1978: Bulk characteristics of heat transfer in the unstable, baroclinic atmospheric boundary layer. *Bound.-Layer Meteor.*, **15**, 399 - 421.
- Gill, A. E., 1968: Similarity theory and geostrophic adjustment. *Quart. J. Roy. Meteor.*
- Hess, G. D., 1973: On Rossby-number similarity theory for a baroclinic planetary boundary layer. *J. Atmos. Sci.*, **30**, 1722 - 1723.
- Hogstrom, U., 1988: Non-dimensional wind and temperature profiles in the atmospheric surface layer: A re-evaluation. *Bound.-Layer Meteor.*, **42**, 55 - 78.
- Izumi, Y., 1971: *Kansas 1968 Field Program data Report*. AFCRL-TR-76-0038. Massachusetts. 79 pp.
- Kader, B. A., 1988: Three-layered structure of an unstably stratified atmospheric surface layer. *Bull. Acad. Sci. USSR, Atmos. and Ocean. Physics*, **24**, 907 - 918.
- Kader, B. A. and A. M. Yaglom, 1990: Mean fields and fluctuation moments in unstably stratified turbulent boundary layer. *J. Fluid Mech.*, **212**, 637 - 667.
- Kader, B. A. and V. G. Perepelkin, 1984: Wind speed and temperature profiles in the atmospheric surface layer in the presence of neutral and unstable stratification. *Bull. Acad. Sci. USSR, Atmos. and Ocean. Physics*, **20**, 112 - 119.
- Kader, B. A. and V. G. Perepelkin, 1989: Effect of unstable stratification on the wind speed and temperature profiles in the surface layer. *Bull. Acad. Sci. USSR, Atmos. and Ocean. Physics*, **25**, 583 - 588.
- Kai, K., 1982: The budget of turbulent energy measured at the ERC 30-m meteorological tower. *J. Meteor. Soc. Japan*, **60**, 1117 - 1131.
- Kaimal, J. C., J. C. Wyngaard, D. A. Haugen, O. R. Cote and Y. Izumi, 1976: Turbulence structure in the convective boundary layer. *J. Atmos. Sci.*, **33**, 2152 - 2169.
- Korrell, A., H. A. Panofsky and R. J. Rossi, 1982: Wind profiles at the Boulder tower. *Bound.-Layer Meteor.*, **22**, 295 - 312.
- Lettau, H., 1979: Wind and temperature profile prediction for diabatic surface layers including strong inversion cases. *Bound.-Layer Meteor.*, **17**, 443 - 464.
- Lumley, L. L. and H. A. Panofsky, 1964: *The Structure of Atmospheric Turbulence*. John Wiley Interscience, New York. 239 pp.
- Melgarejo, J. W. and J. W. Deardorff, 1974: Stability functions for the boundary-layer resistance laws based upon observed boundary layer heights. *J. Atmos. Sci.*, **31**, 1324 - 1333.
- Monin, A. S. and A. M. Obukhov, 1954: Basic turbulent mixing laws in the atmosphere near the ground. *Trudy Geofiz. Inst. Akad. Nauk SSSR*, **24**, 163 - 187.
- Oncley, S. P., C. A. Friehe, J. C. Larue, J. A. Businger, E. C. Itswere and S. S. Chang, 1996: Surface layer fluxes, profiles, and turbulence measurements over uniform terrain under near-neutral conditions. *J. Atmos. Sci.*, **53**, 1029 - 1044.
- Panofsky, H. A., 1961: An alternative derivation of diabatic wind profiles. *Quart. J. Roy. Meteor. Soc.*, **87**, 109 - 110.
- Panofsky, H. A., 1978: Matching in the convective planetary boundary layer. *J. Atmos. Sci.*, **35**, 272 - 276.
- Panofsky, H. A., A. K. Blackadar and G. E. McVehil, 1960: The diabatic wind profile. *Quart. J. Roy. Meteor. Soc.*, **86**, 495 - 503.
- Paulson, C. A., 1970: The mathematical representation of wind speed and temperature profiles in the unstable atmospheric surface layer. *J. Appl. Meteor.*, **9**, 857 - 861.
- Priestley, C. H. B., 1955: Free and forced convection in the atmosphere near the ground. *Quart. J. Roy. Meteor. Soc.*, **81**, 139 - 143.
- Pruitt, W. O., D. L. Morgan and F. J. Lourence, 1973: Momentum and mass transfer in the surface boundary layer. *Quart. J. Roy. Meteor. Soc.*, **99**, 370 - 386.
- Reynolds, O., 1895: IV. On the dynamical theory of incompressible viscous fluids and the determination of the criterion [for turbulence]. *Phil. Trans. of the Roy. Soc. of London (A.)* Vol. **186** - Part 1, 123-164.
- Santoso, E., 1997: A vertical wind profile relationship in the convective boundary layer. *J. Iptek Iklim dan Cuaca*, **1**, 1 - 11.
- Santoso, E. and R. Stull, 1998: Wind and temperature profiles in the radix layer, the bottom fifth of the convective boundary layer. *J. Appl. Meteor.*, **37**, 545 - 558.
- Sheppard, P. A., 1958: Transfer across the earth's surface and through the air above. *Quart. J. Roy. Meteor. Soc.*, **84**, 205 - 224.
- Sorbjan, Z., 1989: *Structure of the Atmospheric Boundary Layer*. Prentice Hall, Englewood Cliffs, New Jersey. 317 pp.
- Stull, R. B., 1988: *An Introduction to Boundary*

- Layer Meteorology*. Kluwer Academic Publishers, Dordrecht, The Netherlands. 666 pp.
- Stull, R. B., 1991: Static stability — an update. *Bull. Amer. Meteor. Soc.*, **72**, 1521 - 1529.
- Stull, R. B., 1997: Reply (to comments on "A convective transport theory for surface fluxes"). *J. Atmos. Sci.*, **54**, 579.
- Sudararajan, A., 1975: Significance of the neutral height scale for the convective, barotropic planetary boundary layer. *J. Atmos. Sci.*, **32**, 2285 - 2287.
- Sugita, M. and W. Brutsaert, 1992: The stability functions in the bulk similarity formulation for the unstable boundary layer. *Bound.-Layer Meteor.*, **61**, 65 - 80.
- Sutton, O.G., 1953: *Micrometeorology, A Study of Physical Processes in the Lowest Layers of the Earth's Atmosphere*. McGraw-Hill. 333 pp.
- Swinbank, W. C., 1968: A comparison between predictions of dimensional analysis for the constant-flux layer and observations in unstable conditions. *Quart. J. Roy. Meteor. Soc.*, **94**, 460 - 467.
- Taylor, R. J., 1960: Similarity theory in the relation between fluxes and gradients in the lower atmosphere. *Quart. J. Roy. Meteor. Soc.*, **86**, 67 - 78.
- Webb, E. K., 1970: Profile relationships: the log-linear range, and extension to strong stability.
- Yaglom, A. M., 1977: Comments on wind and temperature flux-profiles relationships. *Bound.-Layer Meteor.*, **11**, 89 - 102.
- Yamada, T., 1976: On the similarity functions A, B and C of the planetary boundary layer. *J. Atmos. Sci.*, **33**, 781 - 793.
- Yamamoto, G., 1959: Theory of turbulent transfer in non-neutral conditions. *J. Meteor. Soc. Japan*, **37**, 60 - 70.
- Yamamoto, G., 1975: Generalization of the KEYPS formula in diabatic condition and related discussion on the critical Richardson number. *J. Meteor. Soc. Japan*, **53**, 189 - 195.
- Zilitinkevich, S. S., 1969: On the comparison of the basic parameters of the interaction between the atmosphere and the ocean. *Tellus*, **21**, 17 - 24.
- Zilitinkevich, S. S., 1991: *Turbulence Penetrative Convection*. Avebury technical, Aldershot. 179 pp.
- Zilitinkevich, S. S., A. Grachev and J. C. R. Hunt, 1997: Non-local heat and mass transfer in the shear-free convective boundary layer. (Submitted to *J. Fluid Mech.*).
- Zilitinkevich, S. S., A. Grachev and J. C. R. Hunt, 1998: Surface frictional processes and non-local heat / mass transfer in the shear-free convective boundary layer. In *Buoyant Convection in Geophysical Flows*. (Ed. E. J. Plate, E. E. Fedorovich, D. X. Viegas and J. C. Wyngaard). Kluwer academic Publishers, Dordrecht, The Netherlands, 83 - 113.
- Zilitinkevich, S. S. and D. V. Chalikov, 1968a: On the determination of the universal wind and temperature profiles in the surface layer of the atmosphere. *Bull. Acad. Sci, USSR, Atmos. and Ocean. Physics*, **4**, 294 - 302.
- Zilitinkevich, S. S. and D. V. Chalikov, 1968b: The law of resistance and of heat and moisture exchange in the interaction between the atmosphere and an underlying surface. *Bull. Acad. Sci, USSR, Atmos. and Ocean. Physics*, **4**, 438 - 441.
- Zilitinkevich, S. S. and J. W. Deardorff, 1974: Similarity theory for planetary boundary layer of time-dependent height. *J. Atmos. Sci.*, **31**, 1449 - 1452.

## DATA PENULIS



**EDI SANTOSA**, lahir di Jakarta, 12 Maret 1960. Menyelesaikan S1 jurusan Geofisika & Meteorologi di ITB, 1985, menyelesaikan S2 di University of Wisconsin-Medison, USA, 1993. Menyelesaikan S3 di University of British Columbia, Vancouver, Canada, 1999. Bekerja di UPT Hujan Buatan BPP Teknologi sejak tahun 1986, sebagai Research Assistant di University of Wisconsin-Medison, 1994-1995, di University of British Columbia, 1996-1999. Sebagai Teaching Assistant di University of British Columbia, 1998-1999.